

Objectives

- Use the fundamental Theorem of Algebra and its corollary to write a polynomial equation of the least degree with given roots.
- Identify all of the roots of a polynomial equation.

The following statements are equivalent:
A real number r is a root of the polynomial equation $P(x) = 0$
$P(r) = 0$
r is an x-intercept of the graph $P(x)$
$x - r$ is a factor of $P(x)$
When you divide the rule for $P(x)$ by $x - r$, the remainder is 0.
r is a zero of $P(x)$.

Example 1 Writing Polynomial Functions Given Zeros

Write the simplest polynomial function with zeros $-3, \frac{1}{2}$, and 1.

Try it

a) Write the simplest polynomial function with zeros $-2, 2$, and 4.

b) Write the simplest polynomial function with zeros $0, \frac{2}{3}$, and 3.

The Fundamental Theorem of Algebra

Every polynomial function of degree $n \geq 1$ has at least one zero, where a zero Maybe a complex number.

Corollary: Every polynomial function of degree $n \geq 1$ has exactly n zeros, Including multiplicities.
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Example 2 **Finding All Roots of a Polynomial Equation**

Solve $x^4 + x^3 + 2x^2 + 4x - 8 = 0$ by finding all roots.

How many roots?

Step 1 Use the Rational Root Theorem to identify possible rational roots.

Step 2 Graph $y = 4x^4 - 21x^3 + 18x^2 + 19x - 6$ to find the x -intercepts.

Step 3 Test the possible rational roots.

Step 4 Solve for the remaining roots.

Try it!

Solve $x^4 + 4x^3 - x^2 + 16x - 20 = 0$ by finding all roots.

How many roots?

Step 1 Use the Rational Root Theorem to identify possible rational roots.

Step 2 Graph $y = x^4 + 4x^3 - x^2 + 16x - 20$ to find the x -intercepts.

Step 3 Test the possible rational roots.

Step 4 Solve for the remaining roots.

Complex Conjugate Root Theorem

If $a + bi$ is a root of a polynomial equation with real-number coefficients, then $a - bi$ is also a root.

Example 3 Writing a Polynomial Function with Complex Zeros

Write the simplest polynomial function with zeros $1 + i$, $\sqrt{2}$, and -3 .

Step 1 Identify all roots.

Step 2 Write the equation in factored form.

Step 3 Multiply

Try it! Write the simplest polynomial function with zeros $1 + \sqrt{2}$, $2i$, and 3 .

Step 1 Identify all roots.

Step 2 Write the equation in factored form.

Step 3 Multiply