

Objectives

- Identify the multiplicity of roots
- Use the Rational Root Theorem and The irrational Root Theorem to solve polynomial equations

Vocabulary

- multiplicity

Example 1 Using Factoring to Solve Polynomial Equations

Solve each polynomial equation by factoring.

a) $3x^5 + 18x^4 + 27x^3$

b) $x^4 - 13x^2 = -36$

Try it Solve each polynomial equation by factoring.

a) $2x^6 - 10x^5 - 12x^4 = 0$

b) $x^3 - 2x^2 - 25x = -50$

Example 2 **Identifying Multiplicity**

Identify the roots of each equation. State the multiplicity of each root.

a) $x^3 - 9x^2 + 27x - 27 = 0$

b) $-2x^3 - 12x^2 + 30x + 200 = 0$

Try it!

Identify the roots of each equation. State the multiplicity of each root.

a) $x^4 - 8x^3 + 24x^2 - 32x + 16 = 0$

b) $2x^6 - 22x^5 + 48x^4 + 72x^3 = 0$

Rational Root Theorem

If the polynomial $P(x)$ has integer coefficients, then every rational root of the polynomial equation $P(x) = 0$ can be written in the form $\frac{p}{q}$, where p is a factor of the constant term of $P(x)$ and q is a factor of the leading coefficient of $P(x)$.

Irrational Root Theorem

If the polynomial $P(x)$ has rational coefficients and $a + b\sqrt{c}$ is a root of the polynomial equation $P(x) = 0$, where a and b are rational and \sqrt{c} is irrational, then $a - b\sqrt{c}$ is also a root of $P(x) = 0$.

Example 3 Identifying All of the Real Roots of a Polynomial Equation

Identify all the real roots of $4x^4 - 21x^3 + 18x^2 + 19x - 6 = 0$.

Step 1 Use the Rational Root Theorem to identify possible rational roots.

Step 2 Graph $y = 4x^4 - 21x^3 + 18x^2 + 19x - 6$ to find the x -intercepts.

Step 3 Test the possible rational roots $+2$ and $-\frac{3}{4}$

Step 4 Solve for the remaining roots.

Try it! Identify all the real roots of $2x^3 - 3x^2 - 10x - 4 = 0$.

Step 1 Use the Rational Root Theorem to identify possible rational roots.

Step 2 Graph $y = 2x^3 - 3x^2 - 10x - 4$ to find the x -intercepts.

Step 3 Test the possible rational roots

Step 4 Solve for the remaining roots.