Algebra II **Section 5.2**
Auch **Date:**

Objectives

- Define, identify, and graph quadratic functions.
- Identify and use maximums and minimums of quadratic functions to solve problems.

Vocabulary

Axis of symmetry
Standard form-
Minimum value-
Maximum value-

Example 1

Identify the axis of symmetry for the graph of $f(x) = 2(x+2)^2 - 3$

Rewrite the function to find the value of *h.* Use $f(x) = a(x-h)^2 + k$

$f(x) = 2(x-(-2))^2 - 3$

Because $h = -2$, the axis of symmetry is the vertical line $x = -2$

Try it!

Identify the axis of symmetry for the graph of $f(x) = (x-3)^2 + 1$

Properties of a Parabola

For $f(x) = ax^2 + bx + c$, where a,b,and c are real numbers and $a \neq 0,$ the parabola has these properties:

The parabola opens upward if $a > 0$ and downward if $a < 0$.

The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

The vertex is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

The y-intercept is *c*.

Example 2 Graphing Quadratic Functions in Standard Form.

Consider the function $f(x) = x^2 - 4x + 6$

a. Determine is the graph opens upward or downward.

Because *a* is positive, the parabola opens upward.

b. Find the axis of symmetry.

The axis of symmetry is given by $x = -\dfrac{b}{2a}$

$x = -\dfrac{(-4)}{2(1)} = 2$ The axis of symmetry is the line $x = 2$

c. Find the vertex

The vertex lies on the axis of symmetry, so the x-coordinate is 2.

The y-coordinate is the value of the function at this x-value, or $f(2)$.

$f(2) = (2)^2 - 4(2) + 6 = 2$, **the vertex is** $(2,2)$.

d. Find the y-intercept.

Because $c = 6$, the y-intercept is 6.

e. Graph the function.

Graph by sketching the axis of symmetry and then plotting the vertex and the intercept point, $(0,6)$. Use axis of symmetry to find another point on the parabola. Notice that $(0,6)$ is 2 units left of the axis of symmetry. The point on the parabola symmetrical to $(0,6)$ is 2 units to the right of the axis at $(4,6)$.

Example 2b Graphing Quadratic Functions in Standard Form.

Consider the function $f(x) = -4x^2 - 12x - 3$

a. Determine is the graph opens upward or downward.

Because *a* is negative, the parabola opens downward.

b. Find the axis of symmetry.

The axis of symmetry is given by $x = -\dfrac{b}{2a}$

$x = -\dfrac{(-12)}{2(-4)} = -\dfrac{3}{2}$ The axis of symmetry is the line $x = -\dfrac{3}{2}$

c. Find the vertex

The vertex lies on the axis of symmetry, so the x-coordinate is $-\dfrac{3}{2}$.

The y-coordinate is the value of the function at this x-value, or $f\left(-\dfrac{3}{2}\right)$.

$f\left(-\dfrac{3}{2}\right) = -4\left(-\dfrac{3}{2}\right)^2 - 12\left(-\dfrac{3}{2}\right) - 3 = 6$, **the vertex is** $\left(-\dfrac{3}{2}, 6\right)$.

d. Find the y-intercept.

Because $c = -3$, the y-intercept is -3.

e. Graph the function.

Graph by sketching the axis of symmetry and then plotting the vertex and the intercept point, $(0,-3)$. Use axis of symmetry to find another point on the parabola. Notice that $(0,-3)$ is 1.5 units right of the axis of symmetry. The point on the parabola symmetrical to $(0,-3)$ is 1.5 units to the left of the axis at $(-3,-3)$.

Try it!

For each function, (a) Determine is the graph opens upward or downward, (b) Find the axis of symmetry, (c) Find the vertex, (d) Find the y-intercept, (e) Graph the function.

$f(x) = -2x^2 - 4x$

$g(x) = x^2 + 3x - 1$

Example 3 Finding Minimum and Maximum Values

Find the minimum or maximum value of $f(x) = 2x^2 - 2x + 5$. The state the domain and the range of the function.

Step 1. Determine whether or not the function has a minimum or maximum value. Because *a* is positive, the graph opens upward and has a minimum value.

Step 2. Find the x-value of the vertex.

$$x = -\frac{b}{2a} = -\frac{(-2)}{2(2)} = \frac{2}{4} = \frac{1}{2}$$

Step 3 Then find the y-value of the vertex, $f\left(-\frac{b}{2a}\right)$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 5 = 4\frac{1}{2}$$

The minimum value is 4.5. The domain is all real numbers. The range is all real numbers greater than or equal to 4.5, or $\{y | y \geq 4.5\}$.

Try it! **Find the minimum or maximum value of $f(x) = x^2 - 6x + 3$. The state the domain and the range of the function.**

Try it! **Find the minimum or maximum value of $g(x) = -2x^2 - 4$. The state the domain and the range of the function.**