

Objectives

- Define, identify, and graph quadratic functions.
- Identify and use maximums and minimums of quadratic functions to solve problems.

Vocabulary

Axis of symmetry
Standard form-
Minimum value-
Maximum value-

Example 1

Identify the axis of symmetry for the graph of $f(x) = 2(x + 2)^2 - 3$

Rewrite the function to find the value of h . Use $f(x) = a(x - h)^2 + k$

$$f(x) = 2(x - (-2))^2 - 3$$

Because $h = -2$, the axis of symmetry is the vertical line $x = -2$

Try it!

Identify the axis of symmetry for the graph of $f(x) = (x - 3)^2 + 1$

Properties of a Parabola

For $f(x) = ax^2 + bx + c$, where $a, b,$ and c are real numbers and $a \neq 0$, the parabola has these properties:

The parabola opens upward if $a > 0$ and downward if $a < 0$.

The axis of symmetry is the vertical line $x = -\frac{b}{2a}$.

The vertex is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

The y-intercept is c .

Example 2 Graphing Quadratic Functions in Standard Form.

Consider the function $f(x) = x^2 - 4x + 6$

a. Determine if the graph opens upward or downward.

Because a is positive, the parabola opens upward.

b. Find the axis of symmetry.

The axis of symmetry is given by $x = -\frac{b}{2a}$

$$x = -\frac{(-4)}{2(1)} = 2 \quad \text{The axis of symmetry is the line } x = 2$$

c. Find the vertex

The vertex lies on the axis of symmetry, so the x-coordinate is 2.

The y-coordinate is the value of the function at this x-value, or $f(2)$.

$$f(2) = (2)^2 - 4(2) + 6 = 2, \quad \text{the vertex is } (2,2).$$

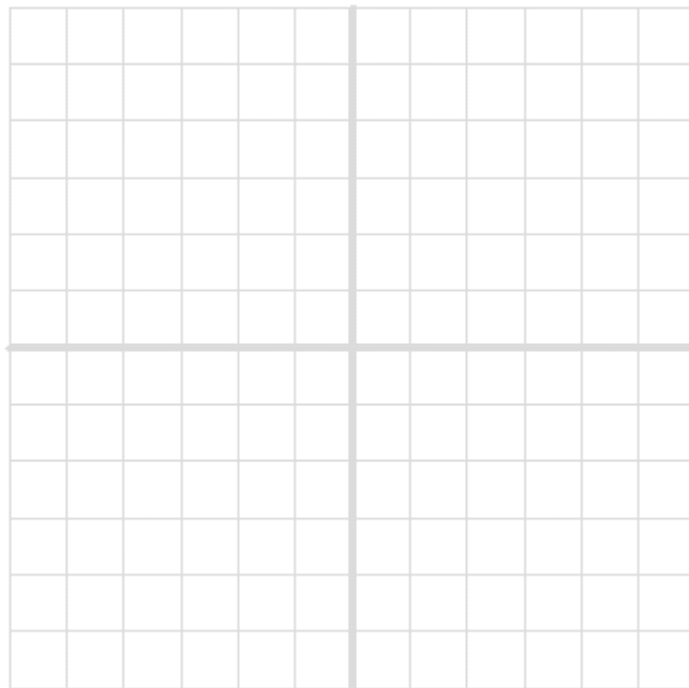
d. Find the y-intercept.

Because $c = 6$, the y-intercept is 6.

e. Graph the function.

Graph by sketching the axis of symmetry and then plotting the vertex and the intercept point, $(0,6)$. Use axis of symmetry to find another point on the parabola.

Notice that $(0,6)$ is 2 units left of the axis of symmetry. The point on the parabola symmetrical to $(0,6)$ is 2 units to the right of the axis at $(4,6)$.



Example 2b Graphing Quadratic Functions in Standard Form.

Consider the function $f(x) = -4x^2 - 12x - 3$

a. Determine if the graph opens upward or downward.

Because a is negative, the parabola opens downward.

b. Find the axis of symmetry.

The axis of symmetry is given by $x = -\frac{b}{2a}$

$$x = -\frac{(-12)}{2(-4)} = -\frac{3}{2} \quad \text{The axis of symmetry is the line } x = -\frac{3}{2}$$

c. Find the vertex

The vertex lies on the axis of symmetry, so the x-coordinate is $-\frac{3}{2}$.

The y-coordinate is the value of the function at this x-value, or $f\left(-\frac{3}{2}\right)$.

$$f\left(-\frac{3}{2}\right) = -4\left(-\frac{3}{2}\right)^2 - 12\left(-\frac{3}{2}\right) - 3 = 6, \quad \text{the vertex is } \left(-\frac{3}{2}, 6\right).$$

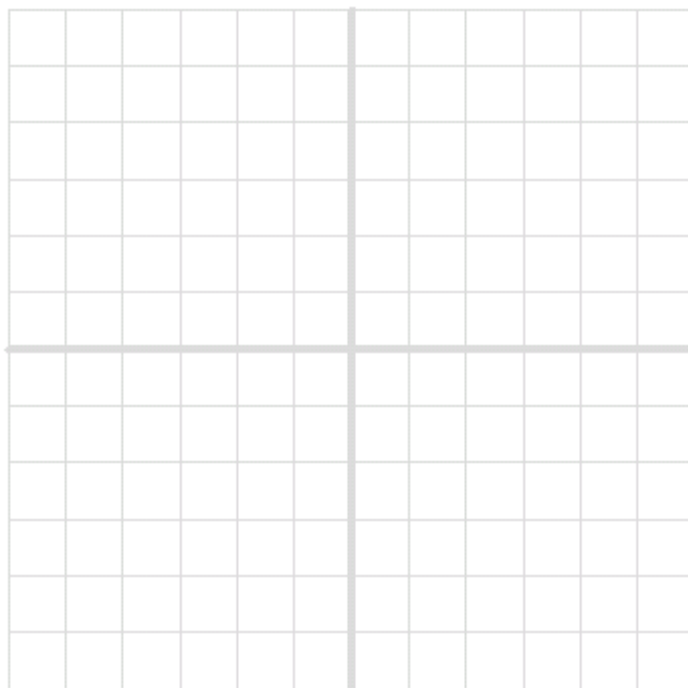
d. Find the y-intercept.

Because $c = -3$, the y-intercept is -3 .

e. Graph the function.

Graph by sketching the axis of symmetry and then plotting the vertex and the intercept point, $(0, -3)$. Use axis of symmetry to find another point on the parabola.

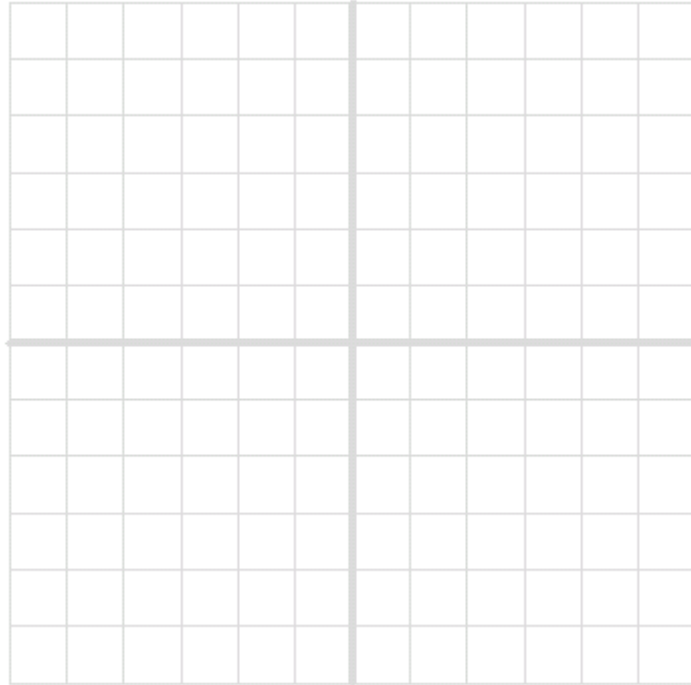
Notice that $(0, -3)$ is 1.5 units right of the axis of symmetry. The point on the parabola symmetrical to $(0, -3)$ is 1.5 units to the left of the axis at $(-3, -3)$.



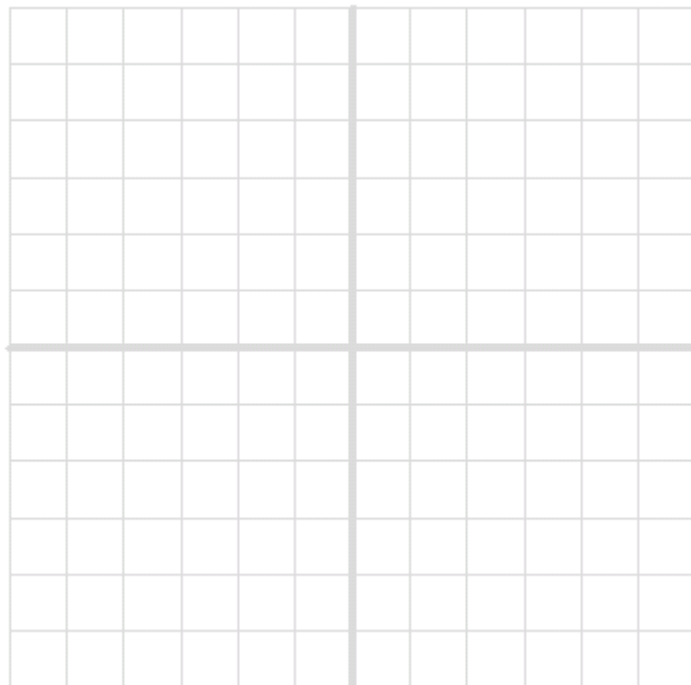
Try it!

For each function, (a) Determine is the graph opens upward or downward, (b) Find the axis of symmetry, (c) Find the vertex, (d) Find the y-intercept, (e) Graph the function.

$$f(x) = -2x^2 - 4x$$



$$g(x) = x^2 + 3x - 1$$



Example 3 Finding Minimum and Maximum Values

Find the minimum or maximum value of $f(x) = 2x^2 - 2x + 5$. The state the domain and the range of the function.

Step 1. Determine whether or not the function has a minimum or maximum value. Because a is positive, the graph opens upward and has a minimum value.

Step 2. Find the x -value of the vertex.

$$x = -\frac{b}{2a} = -\frac{(-2)}{2(2)} = \frac{2}{4} = \frac{1}{2}$$

Step 3 Then find the y -value of the vertex, $f\left(-\frac{b}{2a}\right)$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 5 = 4\frac{1}{2}$$

The minimum value is 4.5. The domain is all real numbers. The range is all real numbers greater than or equal to 4.5, or $\{y | y \geq 4.5\}$.

Try it! Find the minimum or maximum value of $f(x) = x^2 - 6x + 3$. The state the domain and the range of the function.

Try it! Find the minimum or maximum value of $g(x) = -2x^2 - 4$. The state the domain and the range of the function.